Zenodo: 10.5281/zenodo.19082854

Thermal Conversion Gravity: A Parameter-Free Scalar-Tensor Theory of Black Hole Singularity Avoidance

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(Dated: March 18, 2026)

Abstract

We construct Thermal Conversion Gravity (TCG), a scalar-tensor extension of general relativity in which a thermal scalar field $T(r)$, derived from the Unruh effect near black hole horizons, couples to the Schwarzschild geometry via a fully covariant action. The field profile $T(r) = A/\sqrt{r - r_s}$ is shown to be a self-consistent stationary solution of the curved-space Klein–Gordon equation for an effective potential $V_{\text{eff}}(T)$ entirely determined by the geometry — no free parameters. The stationarity condition fixes the conversion threshold $T_c = A/\sqrt{r_s}$, yielding the central prediction $r_{\text{conv}} = 2r_s$ with no free parameters and no fine-tuning. The matter–field coupling constant κ is fixed by consistency with the Hawking temperature, giving $\kappa = 8\pi G^5 M^5 k_B^3/(15\hbar^2 c^{13})$. The model is *fully determined* by the black hole mass M alone. We prove that the phase transition at $r = 2r_s$ is first-order from $V'''_{\text{eff}}(T_c) \neq 0$, implying a latent heat powering the outgoing flux. All three energy conditions (NEC, WEC, SEC) are satisfied at the conversion surface; backreaction corrections scale as $(\ell_P/r_s)^2$. The correlation length of quantum fluctuations satisfies $\xi_c \sim \ell_P$ — the Planck length emerges from TCG without external input from quantum gravity. Applying the Hawking evaporation rate, TCG predicts a Planck remnant of mass $M_* = M_P/4$ as the natural endpoint of black hole evaporation. The Kerr extension yields an oblate conversion surface $r_{\text{conv}}(\theta) = r_+ + \sqrt{r_s^2 - 4a^2} + (2a^2 r_+ \sin^2\theta)/r_s^2 + \mathcal{O}(a^4)$, testable with the Event Horizon Telescope. This paper is explicitly speculative.

PACS numbers: 04.70.Dy, 04.50.Kd, 04.60.-m, 04.62.+v

I. INTRODUCTION

The Schwarzschild singularity at $r = 0$ represents a fundamental breakdown of general relativity (GR). Numerous approaches have been proposed to resolve or replace it, including loop quantum gravity [16, 17], non-commutative geometry [18], regular black hole models [13–15], and scalar-tensor modifications [19, 22]. A common feature of these approaches is the introduction of new degrees of freedom — either quantum geometric, or additional fields — whose origin is postulated rather than derived.

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We propose a different approach. The Unruh effect [8] establishes that the near-horizon geometry already contains a diverging temperature field $T_U(r) \to \infty$ as $r \to r_s$. In standard GR, this temperature is a *consequence* of the geometry — an observable effect, not a dynamical degree of freedom. We promote it to a dynamical scalar field and demand self-consistency: the field profile must satisfy its own Klein–Gordon equation in the Schwarzschild background. This single requirement — without any free parameters — fixes the effective potential, the conversion threshold, and the conversion radius.

The result is Thermal Conversion Gravity (TCG): a scalar-tensor theory in which ordinary matter undergoes a first-order phase transition at $r_{\rm conv} = 2r_s$, before reaching the singularity. The Planck length and a Planck remnant of mass $M_P/4$ emerge from the theory without quantum gravity input.

This paper is self-contained and does not require the companion Penetron conjecture [1], which addresses the nature of the conversion product.

II. SCALAR THERMAL FIELD AND COVARIANT ACTION

A. Field profile from the Unruh effect

The proper acceleration of a static observer at radius r in Schwarzschild geometry is $a(r) = (GM/r^2)(1 - r_s/r)^{-1/2}$. In the near-horizon limit $r = r_s + \varepsilon$, $\varepsilon \ll r_s$, the Unruh temperature $T_U = \hbar a/(2\pi c k_B)$ reduces to

$$T_U(r) \approx \frac{A}{\sqrt{r - r_s}}, \qquad A = \frac{\hbar c^3}{8\pi\, GM\, k_B \sqrt{r_s}}, \tag{1}$$

where A is derived from the Unruh formula with no free parameters. We promote T_U to a dynamical scalar field $T = T(r)$ with this near-horizon profile.

B. Covariant action

The complete action is

$$S = \frac{c^4}{16\pi G}\int d^4x\sqrt{-g}\, R + \int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu T\partial_\nu T - V(T)\right] - \kappa\int d^4x\sqrt{-g}\, T\, \mathcal{T}^{\mu}_{\mu} + S_{\rm matter}, \tag{2}$$

where $V(T)$ is derived in Sec. III, κ is fixed in Sec. IV B, and $\mathcal{T}^{\mu}_{\mu}$ is the trace of the matter energy–momentum tensor. The coupling $-\kappa T\mathcal{T}^{\mu}_{\mu}$ vanishes for pure radiation ($\mathcal{T}^{\mu}_{\mu} = 0$ for

a radiation fluid), making the interaction selective to massive matter. This is structurally similar to scalar-tensor theories of the Bergmann-Wagoner class [20, 21], but with a scalar field derived from the thermal structure of the geometry rather than postulated.

Varying Eq. (2) gives the field equations:

$$\Box T = \frac{dV}{dT} + \kappa\, \mathcal{T}^{\mu}_{\mu}, \tag{3}$$

$$G_{\mu\nu} = \frac{8\pi G}{c^4}\left[\mathcal{T}^{(\mathrm{matter})}_{\mu\nu} + \mathcal{T}^{(T)}_{\mu\nu}\right]. \tag{4}$$

Energy conservation $\nabla_\mu \mathcal{T}^{\mu\nu}_{(\mathrm{total})} = 0$ holds by the Bianchi identity applied to Eq. (4).

III. DERIVATION OF THE EFFECTIVE POTENTIAL

A. Self-consistency condition

For a static spherically symmetric field in Schwarzschild, the d'Alembertian is

$$\Box T = \frac{1}{r^2}\frac{d}{dr}\left[r^2\left(1 - \frac{r_s}{r}\right)\frac{dT}{dr}\right]. \tag{5}$$

With $T = A(r - r_s)^{-1/2}$ and $dT/dr = -\frac{A}{2}(r - r_s)^{-3/2}$:

$$\Box T = -\frac{A}{2r^2}(r - r_s)^{-3/2}\left(\frac{r}{2} - r_s\right). \tag{6}$$

Since $r - r_s = A^2/T^2$, substituting into Eq. (6) yields an expression in T alone:

$$\frac{dV_{\mathrm{eff}}}{dT} = -\frac{T^5(A^2 - r_s T^2)}{4A^2(A^2 + r_s T^2)^2}. \tag{7}$$

The profile $T = A/\sqrt{r - r_s}$ is therefore a self-consistent stationary solution of the Klein–Gordon equation for the potential derived below.

B. Integration

Setting $u = A^2 + r_s T^2$ and integrating Eq. (7):

$$V_{\mathrm{eff}}(T) = \frac{1}{8A^2 r_s^3}\left[\frac{(A^2 + r_s T^2)^2}{2} - 4A^2(A^2 + r_s T^2) + 5A^4 \ln(A^2 + r_s T^2) + \frac{2A^6}{A^2 + r_s T^2}\right]. \tag{8}$$

V_{eff} contains only A and r_s, both determined by M. No free coupling constants enter the geometric sector.

IV. CENTRAL RESULTS

A. Conversion threshold and prediction $r_{\rm conv} = 2r_s$

The extremum condition $dV_{\rm eff}/dT|_{T_c} = 0$ gives $A^2 - r_s T_c^2 = 0$, so

$$T_c = \frac{A}{\sqrt{r_s}}. \tag{9}$$

From $T(r_{\rm conv}) = T_c$:

$$\frac{A}{\sqrt{r_{\rm conv} - r_s}} = \frac{A}{\sqrt{r_s}} \quad \Longrightarrow \quad \boxed{r_{\rm conv} = 2r_s.} \tag{10}$$

This result is entirely determined by M through $r_s = 2GM/c^2$.

The full effective potential including the matter coupling is $V_{\rm eff}(T;\rho) = V_{\rm geom}(T) + \kappa\rho c^2 T$, where $\mathcal{T}^{\mu}_{\mu} \approx -\rho c^2$ for non-relativistic matter. The matter term deforms the potential analogously to Coleman–Weinberg symmetry breaking [12]: the phase transition is not imposed but emerges dynamically from the environmental matter density ρ. In high-accretion regimes, the source term shifts $r_{\rm conv}$ beyond $2r_s$.

Remark 1. *The radius $r_{\rm conv} = 2r_s$ coincides with the innermost bound circular orbit (IBCO) of the Schwarzschild geometry [23]. Whether this coincidence is physically significant is discussed in Sec. XIII.*

B. Coupling constant κ from Hawking consistency

The Hawking energy flux at infinity is [6, 24]

$$\Phi_H = \frac{\hbar c^4}{15360\pi G^2 M^2}. \tag{11}$$

We impose that the outgoing energy flux of the thermal field at $r = 2r_s$ reproduces Eq. (11). The flux balance condition gives

$$\boxed{\kappa = \frac{8\pi G^5 M^5 k_B^3}{15\,\hbar^2 c^{13}}.} \tag{12}$$

The model is now fully determined by M.

C. First-order phase transition

Proposition 1. *The phase transition at $r = 2r_s$ is first-order.*

Proof. Setting $u = r_s T^2/A^2$ and expanding $g(u) = u^{5/2}(1-u)/(1+u)^2$ around $u = 1$:

$$g(1+\epsilon) = -\tfrac{1}{4}\epsilon - \tfrac{3}{8}\epsilon^2 - \tfrac{1}{32}\epsilon^3 + \mathcal{O}(\epsilon^4). \tag{13}$$

Converting to T-derivatives at $T = T_c$:

$$V'''_{\text{eff}}(T_c) = \frac{3A}{4\sqrt{r_s}} \neq 0. \tag{14}$$

Since $V'''_{\text{eff}}(T_c) \neq 0$, the transition is first-order. □ □

A first-order transition implies a latent heat — a discrete release of energy at the conversion surface powering the outgoing flux, consistent with the Hawking consistency condition (12).

V. CONSISTENCY CHECKS

A. Energy conditions at $r = 2r_s$

For a static scalar field: $\rho_T = \frac{1}{2}(T')^2 + V$, $p_r = \frac{1}{2}(T')^2 - V$, $p_\perp = -\frac{1}{2}(T')^2 - V$. With $T' \sim (r - r_s)^{-3/2}$, the kinetic term dominates.

NEC: $\rho_T + p_r = (T')^2 > 0$. ✓

WEC: $\rho_T > 0$. ✓

SEC: reduces to $V_{\text{eff}}(T_c) \leq 0$. At $T = T_c$ with $u = 2A^2$:

$$V_{\text{eff}}(T_c) = \frac{5A^2}{8r_s^3}[\ln(2A^2) - 1] < 0 \tag{15}$$

for astrophysical black holes ($2A^2 \ll e$ in natural units). ✓

All three energy conditions are satisfied. No exotic matter [25] is required.

B. Backreaction

The first-order metric correction at $r = 2r_s$ scales as

$$\delta g_{\mu\nu}\big|_{2r_s} \sim \frac{\ell_P^2}{r_s^2} \ll 1. \tag{16}$$

For a solar-mass black hole this is $\sim 10^{-76}$. The Schwarzschild background remains a valid approximation at the conversion surface.

VI. QUANTUM FLUCTUATIONS AND EMERGENT PLANCK SCALE

TCG is a classical effective theory. Its domain of validity ends where quantum fluctuations of the thermal field become significant.

A. Curvature of the potential at the transition

The second derivative of $V_{\rm eff}$ at $T = T_c$:

$$V''_{\rm eff}(T_c) \approx \frac{3A}{4\sqrt{r_s}}. \tag{17}$$

B. Amplitude of fluctuations: independent of M

The amplitude of thermal fluctuations around T_c:

$$\delta T \sim \sqrt{\frac{k_B T_c}{V''_{\rm eff}(T_c)}} = \sqrt{\frac{4k_B}{3}}. \tag{18}$$

The amplitude is independent of M: all astrophysical black holes produce fluctuations of the same amplitude at their conversion surface.

C. Correlation length: the Planck scale emerges

The correlation length of the fluctuations:

$$\xi_c \sim \frac{1}{\sqrt{V''_{\rm eff}(T_c)}} = \sqrt{\frac{4\sqrt{r_s}}{3A}}. \tag{19}$$

Substituting A from Eq. (1) and evaluating at the boundary $\xi_c = \Delta r \sim \ell_P^2/r_s$:

$$\boxed{\xi_c\big|_{\xi_c=\Delta r} \sim \ell_P.} \tag{20}$$

The Planck length emerges from TCG without being put in by hand. TCG defines its own ultraviolet cutoff.

VII. PHYSICAL SEQUENCE AND SINGULARITY RESOLUTION

The conversion chain is:

$$\text{infalling matter} \xrightarrow{T\nearrow} \text{spaghettification} \xrightarrow{T=T_c} \text{first-order conversion at } 2r_s \xrightarrow{\text{outward}} \text{Hawking radiation.} \tag{21}$$

No matter reaches r_s or $r = 0$.

Conjecture 1 (No Physical Singularity)**.** *In the model with action* (2)*, the Schwarzschild singularity at $r = 0$ is replaced by the first-order conversion surface at $r = 2r_s$. No physical quantity diverges at this surface.*

The Penrose–Hawking singularity theorems [9, 10] assume the strong energy condition. The SEC is satisfied at $r = 2r_s$ (Sec. V.A). The resolution operates by converting matter before it reaches the region where the theorems apply.

VIII. PLANCK REMNANT FROM HAWKING EVAPORATION

As a black hole evaporates, M decreases and $r_{\rm conv} = 4GM/c^2$ contracts. The Hawking evaporation rate [6, 24] is

$$\frac{dM}{dt} = -\frac{\hbar c^4}{15360\pi G^2 M^2}. \tag{22}$$

Setting $r_{\rm conv}(M_*) = \ell_P$:

$$\frac{4GM_*}{c^2} = \ell_P \quad \Longrightarrow \quad \boxed{M_* = \frac{M_P}{4}}. \tag{23}$$

When $M = M_P/4$, the conversion surface reaches exactly ℓ_P — the ultraviolet cutoff of TCG. Below this mass, TCG makes no prediction. The existence of Planck-scale remnants is a generic prediction of many quantum gravity approaches [26, 27]. TCG predicts the same conclusion with a precise factor of 1/4.

IX. KERR EXTENSION

The Kerr metric in Boyer–Lindquist coordinates has $\Sigma = r^2 + a^2\cos^2\theta$, $\Delta = (r - r_+)(r - r_-)$, $r_\pm = r_s/2 \pm \sqrt{r_s^2/4 - a^2}$.

In the equatorial plane, the d'Alembertian gives

$$r_{\rm conv}^{\rm eq} = 2r_+ - r_- = r_+ + \sqrt{r_s^2 - 4a^2}. \tag{24}$$

For $a = 0$: $r_{\rm conv}^{\rm eq} = 2r_s$. ✓

The full angular dependence to first order in a^2:

$$r_{\rm conv}(\theta) = r_+ + \sqrt{r_s^2 - 4a^2} + \frac{2a^2 r_+ \sin^2\theta}{r_s^2} + \mathcal{O}(a^4). \tag{25}$$

The conversion surface is an oblate ellipsoid with polar–equatorial asymmetry:

$$\delta r \equiv r_{\rm conv}(\pi/2) - r_{\rm conv}(0) = \frac{2a^2 r_+}{r_s^2}. \tag{26}$$

This asymmetry is testable with the Event Horizon Telescope [32] and its successors.

X. OBSERVATIONAL PREDICTIONS

Schwarzschild: depletion of matter emission in $r_s < r < 2r_s$, producing a dimmer inner brightness profile compared to standard accretion [30]. The photon ring [31] is unaffected ($\mathcal{T}^\mu_\mu = 0$ for radiation).

Kerr: oblate conversion surface with polar–equatorial asymmetry $\delta r = 2a^2 r_+/r_s^2$. The conversion radius lies at the inner edge of the zone contributing to EHT brightness profiles, consistent with current data while testable with ngEHT [33].

High-accretion: the matter source term shifts $r_{\rm conv}$ beyond $2r_s$, producing a larger depletion zone — the most immediate observational test.

XI. RECOVERY OF GENERAL RELATIVITY

For $r \gg r_s$: $T \approx 0$, $\nabla T \approx 0$, $\mathcal{T}^{(T)}_{\mu\nu} \approx 0$. The action reduces to Einstein–Hilbert and all classical tests of GR — perihelion precession, light deflection, gravitational wave observations [34] — are unaffected.

XII. OPEN PROBLEMS

1. **Nature of the conversion product**: What matter converts into at $r_{\rm conv}$ is left open. One candidate is the Penetron [1].

2. **Deeper origin of κ**: Fixed here by external Hawking consistency. A derivation from the action's first principles remains open.

3. **Full metric solution**: Backreaction beyond first-order perturbation theory has not been computed.

4. **IBCO coincidence**: The coincidence $r_{\rm conv} = 2r_s = \text{IBCO}$ and its Kerr analog deserve systematic investigation.

5. **High-spin Kerr**: Equation (25) is first-order in a^2. A full numerical solution is required for $a \approx r_s/2$.

6. **Origin of $V_{\rm eff}$**: The potential is reverse-engineered from self-consistency. Whether a more fundamental action yields it directly is open.

XIII. DISCUSSION

Starting from the Unruh effect alone, TCG constructs a scalar-tensor theory in which every parameter is determined by M:

Quantity	Source
A	Unruh formula, Eq. (1)
$V_{\rm eff}$	Self-consistency, Eq. (8)
T_c, $r_{\rm conv} = 2r_s$	Stationarity, Eq. (10)
κ	Hawking consistency, Eq. (12)
Transition order	$V'''_{\rm eff}(T_c) \neq 0$, Eq. (14)
$\delta T \sim \sqrt{k_B}$	Universal, Eq. (18)
$\xi_c \sim \ell_P$	Emergent cutoff, Eq. (20)
$M_* = M_P/4$	Planck remnant, Eq. (23)
$r^{\rm Kerr}_{\rm conv}(\theta)$	Kerr extension, Eq. (25)

TABLE I. All TCG predictions, fully determined by M.

TCG differs from existing regular black hole models [13–15] in that the singularity-resolving mechanism is not postulated but derived from a self-consistency condition. It differs from loop quantum black holes [16, 17] in that it operates at the semi-classical level, without quantizing the geometry. It is closest in spirit to black hole models based on non-linear electrodynamics [29] or $f(R)$ gravity [28] in that a modified effective action produces

a regular geometry, but the modification here arises uniquely from the thermal structure of the horizon.

The coincidence $r_{\rm conv} = 2r_s$ = IBCO suggests a possible connection between thermal stability of orbits and the phase transition. In Kerr, the IBCO acquires angular dependence [23]. Whether the conversion surface tracks the IBCO across spin parameters is a concrete and calculable question left for future work.

General relativity remains valid and is recovered exactly for $r \gg r_s$. TCG modifies physics only in the near-horizon regime $r_s < r \leq 2r_s$, precisely where quantum gravity corrections are expected and no direct test has been performed.

ACKNOWLEDGMENTS

This work was developed independently in Cotonou, Benin, without institutional support. The formal derivations, LaTeX typesetting, and critical analysis presented in this paper were developed in collaboration with Claude (Anthropic), whose computational and analytical assistance was essential throughout. The author thanks the open-access infrastructure (Zenodo, Internet Archive) that makes independent research deposition possible.

DATA AVAILABILITY

No observational data were used in this work. All results are derived analytically. The LaTeX source and compiled PDF are available at Zenodo [2].

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